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OPTIMAL PORTFOLIO CONSTRUCTION USING SHARPE'S SINGLE-INDEX MODEL: EVIDENCE FROM CHITTAGONG STOCK EXCHANGE

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Abstract

The study aims to apply Sharpe's single-index model of portfolio construction and evaluate the model's performance on the securities traded on Chittagong Stock Exchange (CSE). For this purpose, the last seven years' daily closing price data of 122 sample securities as well as the daily closing index value of the benchmark market index, CASPI, has been utilized. Sharpe's model smoothens the intricate process of portfolio construction by suggesting a unique number, called the cutoff rate, to measure the desirability of each security's inclusion in the final portfolio. In this study, 38 securities qualified to be a part of the final portfolio, hence, the optimal investment weight for each of them is calculated. An industry-wise analysis reveals that four industries account for about 68 percent of the final portfolio weight. The constructed portfolio yields a daily mean return of 0.1095 percent, which is equivalent to about 49 percent in effective annual terms. The overall portfolio risk, indicated by standard deviation, is found to be only 0.6425 percent. The portfolio beta of 0.3496 also indicates that there is significant nonexistence of systematic risk. An evaluation of the portfolio parameters explicitly reveals that it has outperformed every sample security as well as the market index, in offering the best risk-return combinations, by a large margin. Therefore, the study found Sharpe's model of portfolio construction highly effective in optimizing risk and return in the context of CSE.

Keywords: *Single-Index Model, Sharpe, Optimal Portfolio, Portfolio Management, Chittagong Stock Exchange*

JEL Codes: *G10, G11*

1. Introduction

1.1. Research Background

The pursuit of return always involves some element of risk. Although risk is an inevitable component of any investment, every rational investor strives for minimizing risk and maximizing return at the same time (Paudel and Koirala, 2007). To achieve this often-conflicting goal, one needs to invest in more than one asset. This practice of investing in multiple assets to obtain optimum return with minimum risk is universally known as portfolio construction. The key purpose of portfolio construction is diversification by combining assets (or securities) having less than perfect positive correlations in their returns.

The idea of investment in portfolios is deep-rooted in today's financial world, but this was nonexistent before the late 1960s. Before the advent of portfolio theory, people used to form portfolios but with very different perceptions. At that time, the focus of portfolio investment was to locate the security of well-performing firms at the best price (Beattie, 2019). No one cared about risk until a 25-year grad student in operations research, Harry Markowitz, revolutionized the way we think of portfolio management. Markowitz's pioneering work on "Portfolio Selection" published in 1952 marked the dawn of modern portfolio theory. According to Markowitz (1952), the crux of any portfolio is its risk, not the best price.

The model developed by Markowitz (1952) pertains to forming an optimal portfolio of securities by risk-averse investors. According to his model, a risk-averse investor ought to pick an efficient portfolio of securities, which is defined as the portfolio that maximizes return for a certain level of risk or minimizes risk for a certain level of return. Although Markowitz's work laid the foundation of modern portfolio theory, his model is rarely used in practice (Elton et al., 1976). Because this elegant model required staggering amounts of input, and this made it a time consuming and an expensive solution. If anyone wants to form a portfolio with "N" number of securities, Markowitz's model would require $N \times (N-1)/2$ correlation coefficients (Elton et al., 2009). The magnitude of work and acumen required to form a portfolio using Markowitz's model was well beyond the capacity of all but a few investment professionals, let alone the individual investors. Conceding these limitations, researchers and investment professionals started looking for a portfolio model that simplifies the process and provides an efficient solution. In pursuit of a solution, William F. Sharpe came out with a simplified variant to the Markowitz model in 1963.

Sharpe's model of portfolio formulation, popularly known as Single-Index Model (SIM), is based on the assumption that securities' prices move together because of a common response to market changes (Sharpe, 1963). As per SIM, the return on a broad market index is taken as a proxy for the common macro-economic factor (Bodie et al., 2009). This implies that return on a broad market index is the only macro-economic factor responsible for influencing the systematic portion in a security's return. Sharpe's model smoothens the intricate process of portfolio construction by suggesting a unique number, called the cutoff rate, to measure the desirability of each security's inclusion in the final portfolio. In comparison to Markowitz's model, Sharpe's SIM drastically reduces the parameter estimates needed to form a portfolio. Therefore, this six-decade aged model is still popular among investment professionals and is regarded as one of the effectual ways for portfolio formation.

1.2. Research Objective

Since the advent of SIM, extensive studies have been carried out across the capital markets of developed economies to investigate the model's effectiveness as a tool for optimizing risk and return. However, a few studies have been conducted on the capital markets of developing economies. Besides, none of the studies from developing economies consisted of securities from wide-ranging industrial sectors or covered a longer time horizon. Also, previous studies focused on monthly closing data at best. Data of higher frequency may result in better risk-reward optimization. Owing to the incomprehensiveness of the previous works, studies on developing economies cannot fully reveal the potential of SIM.

Bangladesh, once labeled as a bottomless basket, has become the role model for developing economies. The story of Bangladesh's growth is nothing short of a miracle. According to the World Bank (2019), Bangladesh ranked second among the fastest growing economies in South Asia. Another report by the United Nations (2019), projected a GDP growth rate of 7.4 percent for Bangladesh, which makes the country the third fastest growing economy in the world. Despite the impressive growth potential, the country's capital market is still in its infancy. In 2018, the market capitalization of listed Bangladeshi companies was only 28.242 percent of its GDP (World Bank, 2018). Frequent political unrest, lack of good governance, corruption, and other massive anomalies have been responsible for the poor performance of Bangladesh's capital market (Rahaman et al., 2013). The market is yet to recover from the calamitous crash of 2010, which has significantly affected investors' confidence and led to their massive exodus from the bourses. However, a sound and thriving capital market is a prerequisite if the country wants to fuel and sustain its robust economic growth. This bearish trend of the market poses an enormous challenge for the capital market investors. A well-diversified portfolio can assist them to earn a generous return while keeping the unsystematic risk at bay.

The present study aims to apply Sharpe's SIM of portfolio construction and evaluate the model's performance on the securities traded on Chittagong Stock Exchange (CSE), the port city bourse of Bangladesh. Besides, this study has been carried out to assist the investors as well as investment professionals of CSE to ease the intricate process of portfolio construction, and thereby, optimizing their risk and return. There has not been a single study on the application and evaluation of SIM on the securities traded on CSE. Being the first of its kind, this study presents a unique opportunity to apply the model and assess its performance on CSE.

The rest of the paper is therefore structured as follows. Part 2 gives a review of the literature regarding this issue followed by a brief description of CSE in Part 3. Part 4 focuses on research methodology. Afterward, the research findings are reported in Part 5. Finally, Part 6 concludes the study and offers avenues for future research.

2. Literature Review

The concept of portfolio investment has become vastly ingrained in contemporary theories of investment and finance, but it was non-existent until the 1960s (Beattie, 2019). Before Markowitz's work, investors used to buy securities that have the best potential of earning rewards at the lowest possible price. This heuristic approach of past completely ignored risk diversification and viewed each security investment as a standalone option. According to Markowitz (1952), mean, standard deviation, and correlation with other securities are the key statistics for creating an optimal portfolio. However, his elegant model lacked simplicity, which led successive researchers and investment professionals in the pursuit of a solution. In 1963, William F. Sharpe came up with a simplified variant to the Markowitz model called Sharpe's single-index model (SIM). Since then a lot of researchers and investment professionals weighed the pros and cons of both models and tried to gauge their performance in portfolio optimization.

Bowen (1984) criticized Markowitz's model by comparing it with a useless creature having a prodigious craving for data. He also found the existence of semantic and statistical barriers, which prevent an average investor from coming to grips with this model. Despite its theoretical elegance, Markowitz's model is regarded as impractical and is used less frequently by the investment companies as they are not structured to apply a mean-variance optimization approach (Michaud, 1989). After applying and comparing single-index, multi-index, and constant correlation models of portfolio selection; Elton et al. (1976, 1977, 1978) suggested using SIM for its efficacy and simplicity. In another study, Haugen (1993) found SIM to be applicable in the case of a large population of securities and recommended its use in place of the classical mean-variance approach of portfolio optimization. Paudel and Koirala (2007) applied both Markowitz and Sharpe's model of portfolio construction on a sample of 30 securities traded on the Nepalese stock market. Following the evaluation of both portfolios, they found both solutions to be equally effective in optimizing risk and return. However, they advocated using Sharpe's model because it is more utilitarian in generating an efficient frontier.

In contrary to the previous studies, Briec and Kerstens (2009) found Markowitz's model to be more useful than SIM for long-term investments. They also opined that SIM is unsuitable for optimizing portfolio performance having tenures of multiple periods. Nanda et al. (2010) attempted to integrate three clustering techniques: K-means, SOM, and Fuzzy C-means into portfolio management. In their study, clustering proved to be more time-efficient in asset selection and efficient portfolio identification. Frankfurter et al. (1976) concluded that Sharpe's model is only a simplified solution to the Markowitz's model, and under conditions of certainty, both models produced identical results. However, SIM outperformed its predecessor in portfolio optimization under conditions of uncertainty. Their study also revealed that Markowitz's model performed better when a short span of historical data was used. The study outcome of Omet (1995) is also in line with most of the previous ones. He suggested using either of the models for portfolio investments but preferred SIM for its straightforwardness.

Rani and Bahl (2012) constructed two portfolios on stocks selected from the Bombay Stock Exchange (BSE) using SIM, one allowing short sales while the other prohibiting it. In their study on thirty stocks, both portfolios generated the best risk-return combinations. Nonetheless, the portfolio forbidding short sales did slightly better than the one permitting it. Following the footsteps of the previous study, Sen and Fattawat (2014) constructed two portfolios from thirty stocks traded on BSE, one using the Sharpe's model and the other using Markowitz's model. Their study also recommended using Sharpe's model. Singh and Gautam (2014) also reached a similar conclusion after forming an optimal portfolio of stocks selected from the banking industry of National Stock Exchange (NSE), Mumbai.

In a recent study on Dhaka Stock Exchange (DSE), Mahmud (2019) found SIM to be very effective in diversifying risk and optimizing return. The constructed portfolio of fifty-four equity securities outshined every individual stock under consideration and the benchmark market index (DSEX) in regards to offering the optimum risk-reward combination.

From the literatures reviewed, it is evident that most of the portfolio optimization studies were carried out in the context developed economies. Although a few studies were conducted in the developing ones, none of them were comprehensive enough to unearth the optimization potential of Sharpe's model. Besides, there has not been a single study on the application and evaluation of Sharpe's SIM on the securities traded on CSE. There exists a lacuna of research, which the present study aims to fill-up.

3. A Brief Overview of CSE

The port city bourse of Bangladesh called Chittagong Stock Exchange (CSE) started its operation on October 10, 1995, as a company limited by guarantee and also as a non-profit organization. After the demutualization in 2013, CSE was transformed into a "for-profit" organization (Chittagong Stock Exchange, 2018). Although CSE is about 41 years younger than the country's first stock exchange named Dhaka Stock Exchange (DSE), it has been playing a pioneering role in automating the trading activities and introducing innovative technologies in the capital market. In 2019, CSE reported having 324 listed securities, of which 285 are equity securities, 38 mutual funds, and only 1 corporate bond (Chittagong Stock Exchange, 2019). As revealed by CSE's annual report of 2019, the total market capitalization in FY 2018-19 was BDT 3,293,302 million. Currently, CSE has five market indices: CASPI, CSE50, CSE30, CSI, and CSCX. CSE All Share Price Index (CASPI) is the only index CSE has been maintaining since its inception and is considered as the bourse's benchmark index. In comparison to other major Asia-Pacific markets, CSE is still in a nascent stage. In July 2019, Chittagong Stock Exchange (2019) reported a Market Capitalization to GDP ratio of 12.95 percent, which is considerably lower than that of neighboring stock exchanges. With a fairly inactive as well as under-developed bond market and the absence of derivative products, CSE is an extremely equity-based capital market, which has a long way to go before it can properly cater to the capital requirements of Bangladesh.

4. Research Methodology

4.1. Data Source and Sampling

To construct an optimal portfolio of securities traded on CSE, daily closing price data has been used for the period ranging from March 14, 2012, to March 14, 2019. CSE All Share Price Index (CASPI), has been used as a proxy for the market. Thereby, the daily closing value of CASPI has been used for the same period. The cutoff yield of 91-day Treasury Bill issued on March 18, 2019, has been used as a proxy for the risk-free rate of return (Bangladesh Bank, 2019). The data used in this study is secondary in nature, and are sourced from CSE library and Bangladesh Bank. The sample used in this study were selected based on the following criteria:

Initially, the study aimed at all, precisely 254, "A" category securities listed on CSE. According to Chittagong Stock Exchange (2019), companies that have declared a dividend of 10 percent or more in the last calendar year and are regular in holding annual general meetings (AGMs) are classified as "A" category companies.

Since the study incorporates the data of last seven years, only securities listed on/before March 14, 2012, with CSE are selected in the sample. There were 71 securities that did not meet the criterion, hence excluded from the sample (refer to Table 1).

Finally, only securities offering a positive mean daily return in the aforementioned study period are considered for the sample. Since short selling is not allowed in Bangladesh, securities having negative mean daily returns are excluded, which leads us to a final sample size of 122 securities (refer to Table 1).

Table 1: Industry-wise Representation of Sample Securities

Name of the Industry/Sector	Total Number of Companies	Number of "A" Category Companies	Listed on/before 3/14/2012	Positive Mean Return (Final Sample)	Data Coverage
Bank	29	28	28	10	34.5%
Cement	7	6	5	5	71.4%
Ceramic	5	3	3	1	20.0%
Corporate Bond	1	1	1	1	100.0%
Energy	17	15	11	7	41.2%
Eng. & Electrical	31	22	11	7	22.6%
Foods & Allied	12	8	7	6	50.0%
General Insurance	30	28	27	17	56.7%
ICT	9	7	4	4	44.4%
Leasing & Finance	22	16	15	7	31.8%
Leather & Footwear	6	4	4	4	66.7%
Life Insurance	12	9	9	3	25.0%
Miscellaneous	15	9	7	5	33.3%

Name of the Industry/Sector	Total Number of Companies	Number of "A" Category Companies	Listed on/before 3/14/2012	Positive Mean Return (Final Sample)	Data Coverage
Mutual Funds	38	38	25	23	60.5%
Papers & Printing	5	2	1	1	20.0%
Pharma & Chemical	26	21	13	12	46.2%
Services & Property	7	5	3	2	28.6%
Telecommunication	2	1	1	1	50.0%
Textile & Clothing	50	31	8	6	12.0%
Total	**324**	**254**	**183**	**122**	**37.7%**

Source: Secondary Data Processed, 2019

The final sample of 122 securities, from nineteen industrial sectors, represent 37.7 percent of all securities listed on CSE. To analyze various risk-return characteristics of the sample securities and construct an optimal portfolio using SIM, a number of statistical techniques have been applied. Microsoft Excel 2016, a spreadsheet application, has been used to perform the analyses. As the study incorporates daily closing data of seven years and represents a substantial portion of the population, the likelihood of sampling error and the impact of temporary variations to distort the final outcome have been minimized. Therefore, the sample size can be assumed adequate to make nifty investment decisions.

4.2. Sharpe's Single-Index Model (SIM)

According to Sharpe (1963), the co-movement between securities' return is due to movement in return of a broad market index, which is CASPI in our study. The principal equation underlying SIM is:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i \quad (1)$$

Where, R_i= Expected return on security i, α_i =Intercept of the straight line or alpha co-efficient (Constant), β_i = Slope of straight line or beta co-efficient, R_m = The rate of return on market index, and ε_i = Error term.

The two elements of random variable (α_i) are alpha co-efficient (α_i) and error term (ε_i). Since the error term (ε_i) has an expected value of zero, the mean return on a security can be expressed as:

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m \quad (2)$$

In addition to measurement of return, measurement of risk is also required for optimal portfolio construction. Therefore, the dispersion and co-movement of return are calculated using the following equations:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon i}^2 \quad (3)$$

Where, σ_i^2 = Total variance of a security's return, $\beta_i^2 \sigma_m^2$ = Market related variance, $\sigma_{\varepsilon i}^2$ = Variance of a security's movement that is not associated with the movement of the market index, also known as the security's unsystematic risk.

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \quad (4)$$

Where, σ_{ij} =The covariance of returns between securities i and j, β_i = Beta of security i, β_j = Beta of security j, and σ_m^2 = Variance of market return.

The daily return of sample securities traded on CSE is measured using the following equation:

$$R_{it} = \frac{P_{it}}{P_{it-1}} - 1 \quad (5)$$

Where, R_{it} = Daily return on security i at time t, P_{it} = Daily closing price of the security i at time t, and P_{it-1} = Daily closing price of the security i at time t-1.

Likewise, the daily return of the CSE benchmark market index, CASPI, is measured applying the following equation:

$$R_{mt} = \frac{I_{it}}{I_{it-1}} - 1 \quad (6)$$

Where, R_{mt} = Daily return on CASPI at time t, I_{it} = Daily closing CASPI value at time t, and I_{it-1} = Daily closing CASPI value at time t-1.
Beta coefficient (β_i) is used to measure the sensitivity of a security's return to movement in market's return. It is also called the measurement of a security's systematic risk and calculated as follows:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \quad (7)$$

Where, σ_{im} = Covariance of the security i return with the market return, and σ_m^2 = Variance of the market return.
The risk premium expected by investors from their investment in a risky asset, such as securities traded on CSE, is also known as excess return. The excess return can be found by subtracting the risk-free rate from each security's expected return. The risk-free rate assumed in this study was 3.53 percent per annum, which was taken from the cutoff yield of a 91-day Treasury Bill issued on 18th March 2019 (Bangladesh Bank, 2019).

4.2.1. The Formulation of Optimal Portfolio

Using the mathematical equations portrayed above, it will require a few more steps to create our optimal portfolio. Sharpe's SIM greatly simplified the optimization process by taking a single value to measure the desirability of any security's inclusion in the portfolio. According to SIM, a security's desirability depends on its excess return to beta ratio (Elton et. al., 2009). The mathematical equation of excess return to beta ratio is:

$$\frac{\overline{R}_i - R_f}{\beta_i} \quad (8)$$

Where, $\overline{R}_i$ = Expected return on stock *i*, R_f = Return on a riskless asset, and β_i = Beta coefficient on security i. The additional return on a security for each unit of market specific risk is measured by this ratio.
Following the calculation of excess return to beta ratio, each security under consideration is ranked in descending order of their corresponding ratio to measure their desirability. As short selling is debarred, any security with negative excess return to beta ratio are eliminated from further perusal.
Each security's inclusion in the optimal portfolio is subject to a unique cutoff rate. All securities with excess return to beta ratio above the cutoff rate are included in the optimal portfolio, and the ones below the cuoff rate are excluded. The cuoff point, denoted by C*, is calculated from the properties of all the securities that are a part of the optimal portfolio. If we designate C_i as a representative for C*, its value can be found when *i* securities are assumed to be a part of the optimal portfolio. For a portfolio of *i* stocks C_i is given by:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \frac{(\overline{R}_j - R_f)\beta_j}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}} \quad (9)$$

Where, σ_m^2 = The variance in the market index or CASPI, and σ_{ej}^2 = The variance of a security's movement that is not associated with the movement of market index, which is also referred to as security's unsystematic risk. Following the calculation of C_i for all sample securities, the result of each security is compared with its respective excess return to beta ratio. Under the principle of SIM, all securities used in the computation of cutoff point have an excess return to beta above C_i, and therefore selected in the portfolio. On the other hand, all securities not used in the computation of cutoff point have an excess return to beta below C_i and are eliminated from the portfolio. There will always be one and only one C_i with this characteristic and it is called the cutoff point, C*.
After the selection of securities in the optimal portfolio, the proportion of fund to be invested in each of them for optimizing risk and return is found by:

$$X_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j} \quad (10)$$

Where, $Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{\bar{R}_i - R_f}{\beta_i} - C^*\right)$ (11)

Equation 10 indicates the share of each security in the optimal portfolio and the results add up to one. The subsequent equation, i.e. equation 11, is used to find the relative investment in each security. The residual variance (σ_{ei}^2) has a significant role in its calculation.

4.2.2. Evaluation of Portfolio Performance

Finally, to measure the performance of constructed portfolio, its beta and alpha are estimated. The portfolio beta (β_p) is the weighted average of the individual beta (β_i) of each security incuded in the portfolio, and is denoted by:

$$\beta_p = \sum_{i=1}^{n} X_i \beta_i \quad (12)$$

In the same way, the alpha on the portfolio (α_p) is calculated as:

$$\alpha_p = \sum_{i=1}^{n} X_i \alpha_i \quad (13)$$

Therefore, the portfolio return is found by:

$$\bar{R}_p = \alpha_p + \beta_p \bar{R}_m \quad (14)$$

Lastly, the portfolio risk or the standard deviation (σ_p) is measured by:

$$\sigma_p = \sqrt{(\beta_p^2 \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{ei}^2)} \quad (15)$$

The equations mentioned above (Equation 1 to 15) are sourced from the work of Elton et al. (2009).

5. Findings and Analysis

5.1. Risk-Return Analysis

Risk and return have always been the two crucial determinants of any investment decision. All the portfolio construction models focus on minimizing risk and maximizing return. The application of SIM also requires the estimation of risk and return for each security under consideration. Therefore, daily closing price data of the 122 sample securities and the daily closing value of CASPI have been used to estimate different risk-return characteristics. A summary of the risk-return combinations offered by securities studied is portrayed in Figure 1.

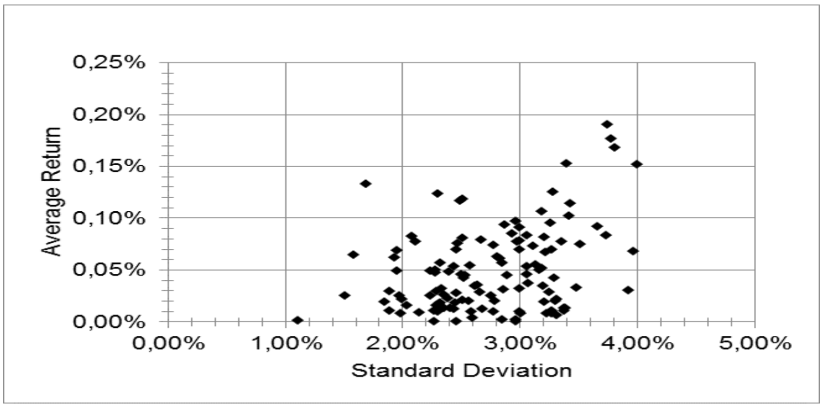

Figure 1: Risk-Return Combinations (Source: Secondary Data Processed, 2019)

An examination of the risk-return combinations in Figure 1 indicates that a typical security yielded a daily mean return of 0.5 percent while carrying a standard deviation of 2.75 percent. Detailed results of various reward and risk indicators such as mean daily return, variance and standard deviation of return, covariance of each security's return with CASPI, and beta coefficient have been presented in Appendix-1.

Analysis of the outcome in Appendix-1 reveals that the security of Monno Ceramic Industries Ltd. offers the highest daily return (0.1910%) followed by Rahima Food Corporation Ltd. (0.1772%) and Legacy Footwear Ltd. (0.1682%). As indicated by the standard deviation, the security of JMI Syringes & Medical Devices Ltd. carries the greatest risk (4%) followed by Standard Insurance Ltd. (3.96%) and MIDAS Financing Ltd. (3.92%). The study also found that 37 securities yielded a return below the market rate of 0.0213 percent. The results of beta coefficient indicate that there are 85 securities that bear a systematic risk below the market.

Application of SIM requires separation of risk and return characteristics into two components: firm-specific part and market-related part. Since market-related characteristics are beyond the control of an investor, this break down will assist us in developing an idea of how well the portfolio will perform. As most of the securities studied (about 70 percent) have a beta coefficient of less than 1 or the market beta, there is a significant possibility of unsystematic risk reduction. But we have to wait until the construction and evaluation of the final portfolio to validate this intuition. Each security's risk and return characteristics in separated form are exhibited in Appendix-2.

An in-depth analysis of the data presented in Appendix-2 shows that for 58 percent of the securities studied, the firm-specific or unsystematic portion was the leading contributor of total return. In terms of risk, firm-specific or unsystematic variance was the principal contributor of the total variance in all of the securities studied. None of the securities had more than half of the variance arising from the systematic portion. These results also strongly suggest that the sample securities present a significant opportunity for risk diversification.

5.2.Ranking Securities

As per the principles of SIM, each security's inclusion in the optimal portfolio depends on its excess return-to-beta ratio. Hence, the excess return-to-beta ratio for each security has been computed and the securities are ranked in descending order of their respective ratios (Appendix-3).

As shown in Appendix-3, the security of Berger Paints Bangladesh Ltd. (Security No. 105) yielded the highest premium for each unit of market-specific or systematic risk. The security of Marico Bangladesh Ltd. (Security No. 109) and British American Tobacco Bangladesh Company Ltd. (Security No. 74) respectively secured the second and third spot in terms of the highest excess return-to-beta generation. Although these securities did not secure the top spots in generating returns, their significantly lower beta value was the reason these companies made their way to the top three in terms of excess return-to-beta ratio. The outcome also shows that about 11 percent of the securities yielded a return below the risk-free rate.

5.3. Setting the Cut-off Point

According to Sharpe (1963), the inclusion of a security in the optimal portfolio is subject to the comparison between excess return-to-beta ratio and a cut-off rate. The securities having excess return-to-beta above the cut-off rate becomes part of the final portfolio, and the ones having lower ratios are ruled out. The following table (Table 2) portrays the variables used for the calculation of the cut-off point.

Table 2: Calculation of Cut-off Point

Rank	Security No.	$\frac{\bar{R}_j - R_f}{\beta_j}$	$\frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\frac{\beta_j^2}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}$	C_i
1	105	0.014503	0.128871	0.128871	8.886114	8.886114	0.000011
2	109	0.007773	0.243163	0.372034	31.283655	40.169769	0.000031
3	74	0.006198	0.884224	1.256258	142.655792	182.825561	0.000102
4	21	0.005263	0.178528	1.434786	33.921199	216.746760	0.000117
5	118	0.004358	0.116659	1.551445	26.769497	243.516257	0.000126
6	75	0.004154	0.307870	1.859315	74.118534	317.634791	0.000150

Rank	Security No.	$\frac{\bar{R}_j - R_f}{\beta_j}$	$\frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\frac{\beta_j^2}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}$	C_i
7	35	0.003609	0.032158	1.891472	8.910686	326.545477	0.000152
8	16	0.003386	0.706823	2.598296	208.722554	535.268031	0.000206
9	30	0.003288	0.265531	2.863827	80.755947	616.023978	0.000225
10	107	0.003030	0.194544	3.058370	64.201490	680.225468	0.000240
11	73	0.002814	0.186833	3.245203	66.390712	746.616180	0.000253
12	68	0.002707	0.385289	3.630492	142.319984	888.936164	0.000280
13	25	0.002499	0.379125	4.009617	151.711835	1040.648000	0.000306
14	27	0.002466	0.737140	4.746757	298.974719	1339.622718	0.000354
15	33	0.002284	0.412006	5.158763	180.393020	1520.015738	0.000379
16	69	0.002014	0.891937	6.050700	442.925453	1962.941191	0.000431
17	66	0.002000	0.173461	6.224161	86.712808	2049.653999	0.000440
18	32	0.001907	0.381693	6.605854	200.183287	2249.837286	0.000461
19	37	0.001786	1.160542	7.766396	649.761363	2899.598649	0.000518
20	76	0.001768	0.403168	8.169564	227.987568	3127.586217	0.000537
21	93	0.001755	0.157192	8.326756	89.592264	3217.178481	0.000544
22	53	0.001691	0.131777	8.458533	77.935441	3295.113923	0.000550
23	113	0.001587	0.430909	8.889442	271.458346	3566.572269	0.000568
24	26	0.001491	0.282465	9.171906	189.456170	3756.028438	0.000579
25	34	0.001443	0.530552	9.702458	367.666338	4123.694776	0.000599
26	104	0.001305	0.591124	10.293583	453.068098	4576.762874	0.000618
27	100	0.001217	0.255562	10.549145	210.002757	4786.765631	0.000625
28	115	0.001146	0.155544	10.704689	135.765456	4922.531088	0.000629
29	2	0.001074	0.671582	11.376271	625.520126	5548.051214	0.000645
30	52	0.001004	0.428041	11.804313	426.174285	5974.225499	0.000654
31	12	0.000882	0.921640	12.725953	1045.311147	7019.536646	0.000666
32	119	0.000822	0.502548	13.228501	611.480235	7631.016881	0.000671
33	59	0.000816	0.807728	14.036229	990.215664	8621.232545	0.000678
34	1	0.000802	1.855008	15.891237	2312.261583	10933.494128	0.000690
35	57	0.000778	1.285717	17.176954	1653.503457	12586.997585	0.000696
36	102	0.000730	0.808258	17.985212	1107.483104	13694.480689	0.000698
37	42	0.000722	0.301587	18.286799	417.546539	14112.027228	0.000698
38	103	0.000713	0.477494	18.764293	669.828272	14781.855500	**0.000698**
39	99	0.000687	0.078851	18.843144	114.767496	14896.622997	0.000698
40	101	0.000663	0.540855	19.383999	815.985853	15712.608850	0.000697
41	87	0.000655	0.237491	19.621491	362.837810	16075.446659	0.000697
42	92	0.000648	0.148359	19.769850	228.976659	16304.423318	0.000696
43	58	0.000636	0.893706	20.663557	1404.803395	17709.226713	0.000694
44	116	0.000631	2.545092	23.208649	4032.468472	21741.695185	0.000686
45	77	0.000621	0.347962	23.556610	560.774892	22302.470077	0.000685
46	36	0.000608	1.633478	25.190089	2684.836674	24987.306751	0.000680

Rank	Security No.	$\frac{\bar{R}_j - R_f}{\beta_j}$	$\frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\frac{\beta_j^2}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}$	C_i
47	117	0.000581	0.595394	25.785482	1025.207766	26012.514516	0.000677
48	122	0.000562	1.089922	26.875404	1940.047384	27952.561901	0.000671
49	67	0.000514	0.790072	27.665476	1538.363110	29490.925011	0.000665
50	91	0.000510	0.027258	27.692734	53.455614	29544.380625	0.000665
51	70	0.000501	0.021004	27.713738	41.919202	29586.299827	0.000665
52	78	0.000494	0.302909	28.016647	613.188882	30199.488709	0.000663
53	61	0.000490	0.968455	28.985102	1976.928519	32176.417228	0.000655
54	5	0.000435	0.498175	29.483278	1143.955858	33320.373086	0.000649
55	56	0.000430	0.901771	30.385048	2094.984695	35415.357781	0.000640
56	13	0.000421	1.223857	31.608905	2906.231979	38321.589761	0.000627
57	90	0.000420	0.703884	32.312789	1676.283467	39997.873227	0.000620
58	110	0.000415	0.764566	33.077355	1840.560623	41838.433851	0.000613
59	22	0.000394	1.366165	34.443520	3467.585010	45306.018860	0.000600
60	31	0.000384	0.990098	35.433618	2581.479695	47887.498555	0.000591
61	84	0.000380	0.626589	36.060208	1648.198806	49535.697361	0.000585
62	11	0.000360	1.132595	37.192803	3143.150279	52678.847640	0.000574
63	79	0.000360	0.544738	37.737541	1511.841470	54190.689110	0.000569
64	94	0.000358	0.282478	38.020019	789.857095	54980.546205	0.000567
65	121	0.000352	0.539071	38.559090	1530.784451	56511.330656	0.000562
66	50	0.000334	0.326067	38.885157	975.867118	57487.197774	0.000559
67	63	0.000331	0.089982	38.975140	271.458581	57758.656355	0.000558
68	45	0.000301	0.300587	39.275727	997.033564	58755.689919	0.000554
69	98	0.000295	0.230652	39.506378	782.506031	59538.195950	0.000552
70	112	0.000295	0.432465	39.938843	1467.704880	61005.900830	0.000546
71	24	0.000254	0.409909	40.348752	1613.745835	62619.646665	0.000540
72	97	0.000243	0.247850	40.596602	1020.468647	63640.115313	0.000536
73	39	0.000239	0.150799	40.747401	630.564818	64270.680131	0.000534
74	72	0.000232	0.062081	40.809482	267.837994	64538.518125	0.000533
75	88	0.000210	0.171998	40.981480	820.557864	65359.075989	0.000529
76	15	0.000192	0.330982	41.312462	1722.045642	67081.121631	0.000522
77	29	0.000182	0.378811	41.691273	2076.784318	69157.905949	0.000513
78	20	0.000174	0.580546	42.271819	3332.774009	72490.679958	0.000500
79	3	0.000163	0.386951	42.658771	2372.539175	74863.219133	0.000491
80	10	0.000150	0.394409	43.053179	2628.933628	77492.152761	0.000481
81	23	0.000148	0.595586	43.648766	4030.792809	81522.945570	0.000466
82	114	0.000111	0.346551	43.995316	3119.383056	84642.328626	0.000455
83	80	0.000109	0.138003	44.133320	1268.713298	85911.041924	0.000450
84	19	0.000107	0.353078	44.486398	3299.017712	89210.059636	0.000439
85	86	0.000107	0.156392	44.642790	1463.281363	90673.340998	0.000434
86	106	0.000101	0.548398	45.191188	5403.649813	96076.990811	0.000418
87	41	0.000100	0.099138	45.290326	991.979361	97068.970172	0.000415
88	120	0.000096	0.226563	45.516889	2364.442066	99433.412238	0.000408
89	60	0.000083	0.341034	45.857923	4085.429698	103518.841936	0.000397

Rank	Security No.	$\frac{\bar{R}_j - R_f}{\beta_j}$	$\frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{(\bar{R}_j - R_f)\beta_j}{\sigma_{ej}^2}$	$\frac{\beta_j^2}{\sigma_{ej}^2}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{\sigma_{ej}^2}$	C_i
90	83	0.000074	0.203282	46.061205	2737.392967	106256.234902	0.000389
91	8	0.000064	0.187947	46.249152	2924.976732	109181.211634	0.000381
92	95	0.000060	0.170403	46.419555	2836.310132	112017.521766	0.000374
93	54	0.000049	0.033453	46.453008	677.420399	112694.942165	0.000372
94	51	0.000041	0.035029	46.488037	850.082585	113545.024750	0.000370
95	96	0.000032	0.072618	46.560655	2249.050032	115794.074781	0.000364
96	64	0.000026	0.044263	46.604918	1691.345047	117485.419828	0.000360
97	18	0.000025	0.131906	46.736824	5249.202465	122734.622293	0.000347
98	44	0.000022	0.007693	46.744517	346.751695	123081.373988	0.000346
99	28	0.000019	0.089576	46.834093	4676.104821	127757.478809	0.000335
100	85	0.000018	0.011525	46.845618	630.309755	128387.788564	0.000333
101	47	0.000016	0.011108	46.856727	714.406976	129102.195540	0.000332
102	6	0.000014	0.035976	46.892703	2582.466595	131684.662135	0.000326
103	49	0.000007	0.005151	46.897854	724.572121	132409.234256	0.000325
104	62	0.000003	0.012767	46.910621	4956.386324	137365.620581	0.000314
105	9	0.000001	0.003610	46.914231	6505.517634	143871.138214	0.000301
106	55	0.000000	-0.000925	46.913306	3337.719902	147208.858116	0.000295
107	4	-0.000007	-0.031484	46.881822	4819.242245	152028.100361	0.000286
108	7	-0.000010	-0.011699	46.870123	1208.546476	153236.646838	0.000284
109	14	-0.000011	-0.049846	46.820277	4589.158859	157825.805697	0.000276
110	38	-0.000015	-0.024602	46.795675	1591.295843	159417.101540	0.000273
111	43	-0.000024	-0.010429	46.785246	427.515386	159844.616926	0.000272
112	48	-0.000047	-0.018849	46.766397	405.111532	160249.728459	0.000271
113	65	-0.000051	-0.149275	46.617122	2908.154234	163157.882693	0.000266
114	40	-0.000081	-0.142794	46.474328	1761.421689	164919.304382	0.000263
115	82	-0.000096	-0.181149	46.293179	1895.666530	166814.970912	0.000259
116	89	-0.000105	-0.137675	46.155504	1309.427077	168124.397988	0.000256
117	81	-0.000232	-0.020461	46.135043	88.031717	168212.429705	0.000256
118	46	-0.000335	-0.021956	46.113087	65.629018	168278.058723	0.000256
119	17	-0.001613	-0.030662	46.082425	19.006577	168297.065300	0.000255
120	71	-0.008226	-0.005713	46.076712	0.694526	168297.759826	0.000255
121	111	-0.022742	-0.108548	45.968164	4.773080	168302.532906	0.000255
122	108	-0.029239	-0.042517	45.925647	1.454120	168303.987026	0.000255

Source: Secondary Data Processed, 2019.

As shown in Table 2, the uppermost Ci value is 0.000698 for Advanced Chemical Industries Ltd. (Security number 103). Therefore, the cut-off rate ($C*$) in this study is 0.000698. Thirty-eight securities have an excess return-to-beta ratio above the cut-off rate, hence, qualified to be a part of the final portfolio. The remaining eighty-four securities with lower ratios are eliminated from the final portfolio. On an average, the securities selected in the final portfolio yielded a daily return of 0.0925 percent and carries a beta coefficient of 0.4765.

5.4. *Constructing the Optimal Portfolio*

As per the guidelines of SIM, the proportion of capital to be invested in each of the thirty-eight securities is calculated and depicted in the following table.

Table 3: Investment proportion of each security in the optimal portfolio

Rank	Security No.	Security Name	Z_i	Weight (X_i)
1	105	Berger Paints Bangladesh Ltd	1.656798	6.8620%
2	109	Marico Bangladesh Limited	1.581526	6.5503%
3	74	British American Tobacco Bangladesh Company Ltd.	3.928036	16.2689%
4	21	Linde Bangladesh Limited	1.365287	5.6547%
5	118	Apex Spinning & Knitting Mills Limited	0.518102	2.1458%
6	75	Jmi Syringes & Medical Devices Ltd.	0.746702	3.0926%
7	35	National Tea Company Limited	0.461344	1.9108%
8	16	Monno Ceramic Industries Ltd.	1.048596	4.3430%
9	30	Rangpur Foundry Ltd.	0.814788	3.3746%
10	107	Kohinoor Chemical Co (Bd) Ltd.	0.612322	2.5361%
11	73	Aramit Limited	0.578060	2.3942%
12	68	Bata Shoe Company (Bd) Limited	1.141843	4.7292%
13	25	Anwar Galvanizing Limited	0.652424	2.7022%
14	27	Eastern Cables Limited	0.911712	3.7761%
15	33	Apex Foods Limited	0.674034	2.7917%
16	69	Legacy Footwear Limited	0.740678	3.0677%
17	66	Apex Footwear Limited	0.346630	1.4357%
18	32	Agricultural Marketing Co Ltd.	0.699789	2.8983%
19	37	Rahima Food Corporation Ltd.	0.754019	3.1230%
20	76	National Polymer Industries Ltd.	0.501134	2.0756%
21	93	Nli First Mutual Fund	0.441619	1.8291%
22	53	Standard Insurance Limited	0.222181	0.9202%
23	113	The Ibn Sina Pharmaceuticals Ltd.	0.505023	2.0917%
24	26	Bangladesh Lamps Limited	0.353801	1.4654%
25	34	Bangas Limited	0.424077	1.7564%
26	104	Ambee Pharmaceuticals Limited	0.439617	1.8208%
27	100	Southeast Bank 1st Mutual Fund	0.294394	1.2193%
28	115	Samorita Hospital Limited	0.164723	0.6822%
29	2	Dutch-Bangla Bank Limited	0.360637	1.4937%
30	52	Sonar Bangla Insurance Ltd.	0.192040	0.7954%
31	12	Heidelberg Cement Bangladesh Ltd.	0.320872	1.3290%
32	119	H.R.Textile Mills Limited	0.083768	0.3469%
33	59	Delta Brac Housing Fin. Corporation Ltd.	0.156246	0.6471%
34	1	Brac Bank Limited	0.263340	1.0907%
35	57	Daffodil Computers Limited	0.115838	0.4798%
36	102	ACI Formulations Limited	0.039412	0.1632%
37	42	Eastern Insurance Company Ltd.	0.015862	0.0657%
38	103	Advanced Chemical Industries Ltd.	0.017158	0.0711%

Rank	Security No.	Security Name	Z_i	Weight (X_i)
		$\sum z_i =$	24.144429	100.00%

Source: Secondary Data Processed, 2019.

As recommended by SIM, the largest investment of capital (16.2689%) should be made in British American Tobacco Bangladesh Company Ltd. and smallest in Eastern Insurance Company Ltd. (0.0657%). Investment of total capital in the aforementioned proportions will assist an investor to achieve the best risk-return combinations possible. An industry-wise classification of the data presented in Table 3 reveals that four industrial sectors account for about 68 percent of total investment weight. A pie chart showing industry-wise investment weight is provided in Figure 2.

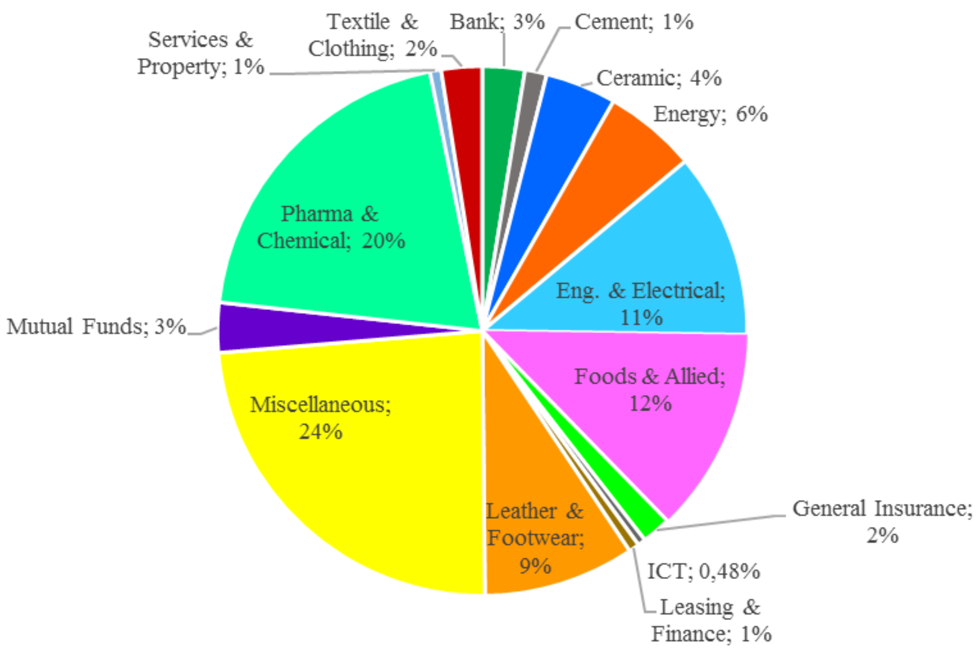

Figure 2: Industry-wise Investment Weight (Source: Secondary Data Processed, 2019)

Although miscellaneous sector occupies the highest investment weight, pharma & chemical industry has the highest number of companies in the final portfolio. The other two industries are food & allied and engineering & electrical occupying 12 percent and 11 percent weight, respectively. Initially, the sample used in this study comprised of securities from 19 industrial sectors, but the final portfolio has securities from 15 industries. No security from telecommunication, papers & printing, corporate bond, and life insurance industry made its way to the final portfolio. The findings of the study also indicate that the final portfolio weight is mostly represented by companies supplying daily necessities.

5.5. Evaluation of Portfolio Performance

Following the selection of securities and their respective weight in the final portfolio, the only task left is to evaluate how the portfolio fared in minimizing risk and maximizing return. Therefore, constructed portfolio's performance has been evaluated by using various risk-return indicators (Refer to Appendix-4). A summary of the key results from Appendix-4 is presented in Table 4.

Table 4: Performance of Constructed Portfolio

Portfolio Return Calculation		**Portfolio Risk Calculation**	
Alpha on Portfolio (α_p)=	0.00102	$\sum_{i=1}^{N} X_i^2 \sigma_{ei}^2 =$	0.0000311702
Beta on Portfolio (β_p)=	0.3496	$\beta_p^2 \sigma_m^2$=	0.0000101121
Return on CASPI $(\overline{R}_m)$=	0.0213%	Portfolio Standard Deviation (σ_p) =	0.6425%
Portfolio Return $(\overline{R}_p)$=	0.1095%	Portfolio Coefficient of Variation (CV) =	5.8688

Source: Secondary Data Processed, 2019.

From the results presented in Table 4, it is evident that the constructed portfolio yields an average return of 0.1095 percent. At a glance the portfolio return may appear insignificant, but we must not forget the fact that this is the daily average return. The effective annual return of the portfolio is about 49.09 percent. At such rate, an investment can grow by its own size within 1.74 years (approximately).

As indicated by the standard deviation, the overall risk of the constructed portfolio is 0.6425 percent, which is significantly lower than the discrete standard deviation of any security studied. In comparison to the market beta, the portfolio beta of 0.3496 indicates that there is significant nonexistence of market specific risk.

An investigation into the result of CV also leads us to similar conclusion. None of the 122 securities studied had a CV as low as or even close to the portfolio CV. An average security in this study had a CV of 220.5751, which is around 38 times bigger than that of the portfolio. All these results indicate that the portfolio formed using SIM offers the best risk-return combinations, i.e. maximum return with minimum risk. Hence, it can be concluded that the portfolio optimization objective has been met and the final portfolio is made of efficient securities.

6. Conclusion, Limitations and Scope for Further Study

6.1. Conclusion

The findings of the study explicitly reveal that the constructed portfolio outperformed every sample security, in offering the best risk-return combinations, by a large margin. The portfolio constructed using Sharpe's SIM indeed diversified risk and yielded the best possible return. The utility of Sharpe's SIM is not limited to its optimization efficiency; the model is highly effective in simplifying the portfolio construction process. In fact, with 122 sample securities, Markowitz's model would require 7,625 pieces of information to construct an optimal portfolio. Sharpe's model did the job with only 368 pieces of information. Hence, we can infer that the five-and-a-half-decade old model propounded by William F. Sharpe works effectually for optimizing the risk and return statistics for the investors of CSE. However, no portfolio optimization model can immune investors forever from changes in economic conditions. Therefore, an investor has to actively evaluate the performance of each security at regular intervals and make necessary amendments in the portfolio accordingly.

6.2. Limitations and Scope for Further Study

The study is subject to a number of limitations. Firstly, the sample consist of securities that belong to Category-A on CSE, hence, future studies can work with securities from other categories. The study also incorporated the daily closing data of last seven years, thereby excluding the securities that were not listed on CSE on/before the study period. Finally, securities offering negative mean returns were eliminated from the sample as short selling is prohibited in Bangladesh. But if it were allowed, the investors could also benefit from selling those securities. Therefore, future studies can form portfolios with negative returns.

As the study focuses on evaluating the effectiveness of Sharpe's SIM on the securities traded on CSE, application of other portfolio optimization models, like Markowitz's or Constant correlation model, could enhance the robustness of the study. According to the principles of SIM, security prices move together because of common co-movement with the market. However, some studies provide evidence that there are influences beyond the market force (Elton et al., 1976). To capture the non-market influences, future studies can also focus on applying multi-index model.

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